XII.—*On the Constants of the Barometric Formulæ which make correct allowance for the Hygrometric State of the Atmosphere.* By Henry Laws Renny, *M. R. I. A., Lieutenant, Royal Engineers, Retired List.*

Read June 14, 1858.

In a paper which I have already submitted to the Royal Irish Academy, and which has been published by the Academy, I have given a formula which makes correct allowance for the hygrometric state of the atmosphere. In this paper I have stated strongly the necessity of a new constant, to be determined by a considerable number of observations, hygrometric as well as barometric, inasmuch as the constant recommended by Laplace (viz., 18336·0 metres = 60158·57 English feet), having been obtained without a systematic and correct consideration of the influence of the vapour of the water of the atmosphere, must of necessity be erroneous.

In submitting my former paper to the consideration of the Academy, I was aware that such paper, without a new constant, was no more than a theory practically useless; but as I had reason to believe the theory sound, having studied it with very great care, I did not hesitate to submit the formula containing it (though incomplete) to the Academy, being aware that at any future period, any person acquainted with the subject, and having opportunity to make the necessary observations, could with facility work out the desired new constant.

Circumstances have enabled me to obtain the new constant, as also much valuable information connected with the subject of barometric observations, which I have now the honour, very respectfully, to submit to the judgment of the Academy.

When, in my former paper, I stated the necessity of obtaining the new

constant by a considerable number of observations, hygrometric as well as barometric, it was my opinion (an erroneous one) that in this way only such constant was to be determined. I made arrangements, therefore, early in the year 1857, for such observations, during my residence at the head of Lake Geneva, near to the Castle of Chillon. In a work of this kind the first thing to be done was evidently to obtain, by means of accurate spirit-levelling, the heights of various stations above a common starting-point, the height of such starting-point above the level of the sea being known, if not with the precision obtained by spirit-levelling, at all events approximately. Both these objects have been attained satisfactorily: the first, by my own personal exertions, working with the very best instruments of London manufacture; the latter, by the labours of civil engineers, French and Swiss, who have connected the level of Lake Geneva with the level of the sea at the Gulf of Lyons, by accurate spirit-levelling.

My own starting-point was the central one of three iron bars, connected with the Limnimetre of Geneva, at the north-west foundation stone of Chillon Castle. My highest point is upwards of 1000 English feet above the foundation stone of Chillon Castle, near to a new hotel, called "Pension Rigi Vaudois," immediately above the village Montreux.

Several intermediate points have been determined by me with peculiar care, and connected with the extreme points of the levelling operations.

It will, I believe, be satisfactory to the Academy to be made acquainted with the peculiar helps and facilities which I enjoyed in my labours near Montreux.

I possessed two excellent mountain barometers, made by Mr. Newman, Regent-street, London; a wet and dry bulb hygrometer, made by Ronketti, Great Russell-street, London; also a Gravatt level of London manufacture; and I was assisted by M. A. Morlott, *ci-devant* Professor of Geology at the College of Lausanne, a young man of high scientific attainments. He also possessed an excellent syphon mountain barometer, together with a wet and dry bulb hygrometer, both of Vienna manufacture.

At the commencement, and for some time afterwards, of the joint labours of M. A. Morlott and of myself, I frankly confess I did not entertain much hope of being able to make a number of observations sufficiently great to be

able to determine the desired new constant as accurately as it was my wish to determine it; when, in examining with care the formula already submitted to the Academy, I perceived that as it takes account of every quantity, however small, of aqueous vapour which may be formed in the atmosphere, it necessarily answers for a state of atmosphere in which the vapour of water becomes nothing; consequently, the desired new constant must be such as belongs to a formula suited to a hypothetic atmosphere consisting of absolutely dry air. Now the constant for such a formula is obtainable from the well-known equation

$$C = \frac{0 \cdot 76}{M \times D} \text{ metres;}$$

in which C represents the desired new constant; M, the modulus of common logarithms; and D the ratio of the specific gravities of dry air and quicksilver, under a pressure of 0·76 metres of quicksilver. For proof of this equation, and the mode of obtaining the required new constant, *vide* Appendix, pp. 658, 659, and 660.

The constant obtained in this manner for latitude 45°, at the level of the sea, at freezing-point = 18404·9 metres = 60384·6 English feet (*vide* Appendix, page 659).

It is worthy of notice, that the constant thus determined from the consideration of the ratio of the specific gravities of dry air and quicksilver, is more correct than a constant obtained from any number, however great, of observations hygrometric and barometric, because, although by a very great number of observations, their unavoidable errors, sometimes over and sometimes under the truth, may be neutralized, or rendered for practical purposes insensible; yet the errors which arise from assuming the arithmetic mean of temperatures, given by the detached thermometers, to be the real mean temperature of the atmospheric column between the stations of observation, cannot be in the least degree diminished by any number of observations, however great. It is, moreover, worthy of special attention, that without a formula, which makes correct allowance for the hygrometric state of the atmosphere, the new constant obtained by means of the equation

$$C = \frac{0 \cdot 76}{M \times D} \text{ metres,}$$

could not be applied in practice; and it is in this point of view that the correct formula, already communicated by me to the Academy, is possessed of greatest interest.

Although the labour undertaken by me, in co-operation with M. A. Morlott, near Montreux, was in no way necessary for obtaining the new constant, yet I do not regret the employment of time on such labour, forasmuch as the observations made at Montreux have been very useful in another direction, which will appear to the Academy before this paper is concluded.

In the midst of my work near Montreux, and after I had made use of the constant 18404·9 metres (obtained from the consideration of the ratio of the specific gravity of dry air to that of quicksilver), in calculating several observations made at that place, M. E. PLANTAMOUR, Professor of Astronomy at the Academy of Geneva, having been apprised by M. Morlott of my efforts to improve the barometric formula, forwarded to me a copy of his quarto, entitled "Resumé des Observations Thermometriques et Barometriques, faites à l'Observatoire de Genève et au Grand-Saint-Bernard, pendant les dix anneés, 1841 à 1850, suivi de tables Hygrometriques, calculées d'aprés la formule de BESSEL, par E. PLANTAMOUR, Professeur d'Astronomie de Genève." M. PLANTAMOUR afterwards sent me a second copy for the Royal Irish Academy, which, with two other tracts, containing very valuable matter, I have deposited in the Library of the Academy.

The importance of these three tracts in reference to the new constant and the new formulæ, which make correct allowance for the hygrometric state of the atmosphere, can be understood only by such persons as may have time and opportunity to peruse them.

I have now to inform the Academy that the constant which I had obtained from the equation

$$C = \frac{0{\cdot}76}{M \times D} \text{ metres,}$$

making use of the experiments of REGNAULT at Paris, differs from the constant obtained by M. E. PLANTAMOUR from the same equation by only the tenth part of a metre (not more than four English inches): the constant of M. PLANTAMOUR being 18404·8 metres, my own being 18404·9 metres. The latter value, viz., 18404·9 metres, had been employed by me in calculations from

my observations, hygrometric and barometric, made near to Montreux, some time before I had heard of M. PLANTAMOUR or his labours in this department of science, making use of my own formula before I had seen or heard anything of BESSEL'S formula. It is satisfactory to reflect that, as to results of calculation, there is a perfect agreement between those of BESSEL'S formula and those of my own, although their forms are strikingly different. The truth of this statement will appear by-and-by, when we apply them to a series of observations, hygrometric and barometric, made during a continuous period of ten years, from 1841 to 1850, both years inclusive, by night as well as by day, at the Convent of the Great Saint Bernard and Observatory of Geneva. This accordance is equally striking when we apply the formulæ to observations made simultaneously at Mont Blanc and the Convent of Great Saint Bernard, and to my own observations made near Montreux in the spring of the year 1857.

Before I present to the Academy such results, I desire to state clearly my reason for believing that the constant recommended by LAPLACE, viz., 18336·0 metres, is altogether too small, and ought to be replaced without hesitation by the new constant, 18404·9 metres. In the first place, the height which it gives for Convent Saint Bernard above Geneva Observatory is less than the true height, ascertained by accurate spirit-levelling, by more than 16·0 metres; whereas, the error by my new constant is little more than 4·0 metres being less than error by LAPLACE'S constant by 12·0 metres. True it is, that part of the error of LAPLACE'S formula is to be attributed to the fact that the allowance for the presence of vapour of water, according to LAPLACE'S unhappy mode of allowing for it, is too small by more than 3 metres; yet, taking this into account, there remain 9 metres of error to be accounted for. There can be no doubt, therefore, that the constant is too small, and indeed this can be proved by the very mode employed by LAPLACE in arriving at it. For if we consult the tract, entitled "Sur la detérmination des Hauteurs par le Barometre par Monsieur E. PLANTAMOUR" (being one of the two tracts presented by me to the Academy), at page 6 we read that LAPLACE, in the calculation of his constant (viz., 18336·0 metres), made use of a set of observations, barometric and thermometric, by RAMOND, in the Pyrenees, made at noon in the fine weather of the summer months.

Now, it is established by experiments which admit of no controversy, that barometric observations made under such circumstances give heights altogether too great, and this agrees with statements made by RAMOND himself, who tells us, in his second memoir, that at the tops of mountains, as also on the plains and in the bottoms of valleys, the barometric observations of the forenoon as of the afternoon give heights so much the less than those made at noon, as the moments in which they are made are the more removed from noon. Now, as LAPLACE'S constant was calculated from observations made at noon in the fine weather of the summer months, which invariably gave heights greater than the true heights, if employed in calculations made with a correct constant, it follows, as a mathematical certainty, that the constant thus determined must be too small; because the error in excess, arising from such observations, is caused by the fact that the arithmetic mean of the temperatures, as given by the detached thermometers, is greater than the real mean temperature of the atmospheric column, situate between the stations of the barometric observation. Now, the well-known formula of barometric calculations is

$$H = C \times \log \frac{B}{B'} \times \left(1 + l \,.\, \frac{t+t'}{2}\right), \quad \text{or } C = \frac{H}{\left(1 + l \,.\, \frac{t+t'}{2}\right) \times \log \frac{B}{B}},$$

in which equations, C represents the constant of the formula; H is assumed as the correct height of one station above the other; B, B' are the true pressures of the atmosphere; l is the expansion of air for one degree of temperatûre, and t, t' are the temperatures as given by the detached thermometers. Now, if in these equations, $\frac{t+t'}{2}$ be too great, it is obvious that C must be too small. Thus we have double testimony to induce us to believe the constant of LAPLACE to be too small.

It is, however, consolatory that in giving up LAPLACE'S constant, it is to make room for another, obtained in a better way, and subject only to such small errors as are inseparable from the mode of obtaining the ratio of the specific gravity of dry air to that of quicksilver. I have not the least hesitation in declaring my belief that the new constant, thus obtained, being subject only to such error, is very much to be preferred to a constant calculated from

even an extremely great number of observations, hygrometric as well as barometric, because of the error arising from assuming the arithmetic mean of temperatures as given by the detached thermometer to be the *real* mean temperature of the atmospheric column between the stations of observation.

For it is to be borne in mind, that this last error in no way resembles the unavoidable small errors of observation, which are sometimes above, sometimes below the mark, and which errors may be rendered (for practical purposes) nearly insensible by using a sufficiently great number of observations. In fact, the error connected with the mean temperature of atmosphere,—when observations by night as well as by day are made,—have the same sign, being minus (−), which has been established as fact by the result of ten years' observations made at the Convent of the Great Saint Bernard and the Observatory of Geneva, when compared with the height of the Convent above the Observatory, obtained by extremely accurate spirit-levelling. This fact will be pointed out to the Academy in a satisfactory manner, before this paper is brought to a conclusion.

I have now to call the attention of the Academy to a small Table, showing the heights of the Convent of the Great Saint Bernard above the Observatory of Geneva, as calculated by the formula of M. E. PLANTAMOUR (which is only a variety of BESSEL'S formula), as calculated by formula of BESSEL, as calculated by my own formula, as calculated by that of LAPLACE, as calculated by that of BAYLEY, the same with that of POISSON (this latter being but a modification of LAPLACE'S). The same Table shows the error of each formula; also the influence of the vapour of water of the atmosphere in adding to the height, as calculated from a formula suitable to an hypothetic atmosphere of simple dry air. The errors shown in this Table have been obtained by comparing the heights, as calculated by the five formulæ, with the true height, ascertained by extremely accurate spirit-levelling.—(*Vide* tract, entitled "Nivellement du Grand-Saint-Bernard, par Messrs. F. BURNIER et E. PLANTAMOUR," being one of the tracts which I have presented to the Academy.)

Let H represent the height of the Convent of the Great Saint Bernard above the Observatory of Geneva, as obtained by calculation from the mean of ten years' observations, hygrometric and barometric, made by night as well as by day, at the Convent and at the Observatory, from the year 1841 to 1850, both

years inclusive (for such observations *vide* M. PLANTAMOUR's quarto, entitled "Resumé," &c., &c., pages 8, 21, 29, and 37 ; also pages 51 and 57).

The true height of the Convent above the Observatory, according to extremely accurate spirit-levelling, is 2070·34 metres.—(*Vide* tract, entitled "Nivellement du Grand-Saint-Bernard," page 10, line 13 from the top of the page).

TABLE I.

True Height of Convent above the Observatory = 2070·34 *metres.*

	Plantamour.	Bessel.	Renny.	Laplace.	Bayley.
	Metres.	Metres.	Metres.	Metres.	Metres.
H, ...	2066·04	2066·49	2066·35	2054·89	2055·34
Errors, ...	− 4·30	− 3·85	− 3·99	− 15·45	− 15·00
Increase of height due to vapour of water,	+ 5·56	+ 6·16	+ 6·06	+ 2·40	+ 2·40

I have now to state, that the slight differences between the heights, as calculated by the formulæ of PLANTAMOUR, of BESSEL, and of RENNY, as shown in the above small Table, arise from the circumstance that these three formulæ do not employ the same value of the elastic force (or tension) of vapour of water,—for with the same data of calculation these three formulæ give the same results,—and seeing that the formula of M. PLANTAMOUR is only a slight variation of that of BESSEL, having a different value of the elastic force of vapour of water, I shall not take the trouble of bringing it forward again, or applying it to any other observations which may be hereafter noticed in this paper. I have now to direct attention to the application of the four formulæ given above to the calculated height of Mont Blanc above the Convent of the Great Saint Bernard, obtained from observations, hygrometric as well as barometric, made by Messrs. BRAVAIS and MARTINS during their ascent of Mont Blanc, 29th August, 1844, compared with observations made at the Convent.—(For the observations made by Messrs. BRAVAIS and MARTINS, *vide* quarto, entitled "Resumé," &c., &c., page 70.)

The height of Mont Blanc above the Convent has not been ascertained either by spirit-levelling, or by the less accurate method of geodetic operations.

The importance of the following small Table, which gives the results of the application of the four formulæ to Mont Blanc and the Convent of Saint Bernard, consists in its pointing out the harmony between the workings of BESSEL's formula and of my own. It also shows in a striking manner the peculiar defect of the formula of LAPLACE, as also that by BAYLEY, in making allowance for the hygrometric state of the atmosphere, when the mean temperature, as given by the detached thermometers, is below the freezing-point, which was the case during the ascent of Mont Blanc by Messrs. BRAVAIS and MARTINS.

The respective heights of Mont Blanc above the Convent of the Great Saint Bernard, as obtained by the four formulæ, from observations by BRAVAIS and MARTINS, compared with observations made at the Convent, are as follow :—

TABLE II.

	Bessel.	Kenny.	Laplace.	Bayley.
	Metres.	Metres.	Metres.	Metres.
H, ...	2339·2	2339·0	2324·7	2323·5
Increase of height due to vapour of water,	+ 5·1	+ 5·0	− 0·6	− 0·6

By this Table it appears that LAPLACE's formula diminishes height in making allowance for influence of vapour of water of the atmosphere, whereas the real operation of the action of vapour of water is of quite an opposite character.

I have now to bring before the Academy a subject of great importance, viz : —The horary correction, without which the calculations for height, made by help of the most correct formulæ, from even an extremely great number of observations, hygrometric as well as barometric, are subject to very serious errors indeed.

In the tract, entitled " Sur la Determination des Hauteurs," &c., M. PLANTAMOUR has given a small table of horary corrections for every second hour of the four seasons (viz., Winter, Spring, Summer, and Autumn). But inasmuch as M. PLANTAMOUR knew not the correct height of the Convent Saint Bernard above the Observatory of Geneva, as since ascertained by accurate spirit-levelling, and seeing that M. PLANTAMOUR has calculated his small table of horary corrections from incorrect data, this praiseworthy attempt of M. PLANTA-

MOUR to accomplish a peculiarly important object must be set aside as defective. This brings me to a table of horary corrections calculated by myself, with data extremely correct, for every hour (night as well as day) of *every* month of the year. My method of calculating this Table is the following:—With my own correct formula and constant, obtained by the consideration of the ratio of specific gravities of dry air and quicksilver, I made no less than two hundred and eighty-eight distinct calculations for every hour, night and day, of every month of the year, from the data supplied by M. PLANTAMOUR's quarto, entitled "Resumé" (*vide* pages 8, 21, 29, 37, 51, and 57). Then subtracting *each* calculated height from the true height, as ascertained by accurate spirit-levelling, viz., 2070·34 metres (*vide* tract, "Nivellement du Grand-Saint-Bernard"), the error of each calculated height is known. Such error, being divided by the calculated height, gives the horary correction—for if such horary correction be multiplied by the calculated height, we necessarily have the difference of calculated and true height; and such difference being added to, or subtracted from the calculated height, according to the sign of the horary correction, necessarily gives the true height. Thus, as unity is to unity, plus or minus the horary correction, so is the calculated height (given by formula) to the true height. I have added to the horary correction, in Table X., the arithmetic means of temperatures, as given by the detached thermometers, which may be found in the quarto, entitled "Resumé," &c., at pages 8 and 29; and by inspection of this Table many striking peculiarities may be seen connecting these means and the errors of calculated heights, some of which, having important bearings on the subject of the present paper, I proceed to notice. By inspection of the said Table of horary corrections and means of temperatures, it appears that the greatest horary corrections, having the minus sign (−), which indicate the greatest errors in excess of calculated heights, take place at one hour P. M.; and that for some months the highest temperatures take place at the same hour, but that for other months the greatest heat takes place at 2 P. M.

It also appears that the greatest errors of calculated heights, in defect, take place after midnight, between 2 and 5 o'clock, and that the connexion between greatest errors in defect and lowest temperatures, occurring during night-time, is by no means so close as the connexion between the greatest errors in excess

and highest temperatures occurring during the day-time. It also appears by inspection, that the very greatest error of calculated height takes place at one hour P. M. of the month of July; this error amounts to 31·14 metres, as may be ascertained by an easy calculation with the data of the Table. The greatest error, in defect, of the entire year takes place in December at five hours after midnight, and amounts to 26·88 metres. The error in defect of the month of July, which takes place from two to three hours after midnight, amounts to 16·7 metres, little more than one-half of the error in excess during the greatest heat of the day of the same month.

This Table also tells us that the horary corrections vary more between the hours of any given month than between the corresponding hours of two consecutive months. It also shows us that the errors in excess are *individually* greater than errors in defect, although the sum of errors in excess is less than sum of errors in defect, which is evident from the fact that the height of the Convent above the Observatory, as calculated from the mean of ten years' observations, by night as by day, is less than true height by nearly 4 metres (*vide* Table I. of this Paper, page 630). It also appears by inspection, that the errors are greatest near 1 o'clock P. M. of summer months, consequently, such moments are the most unfavourable for observations wherewith to calculate a constant; yet these are the moments chosen by LAPLACE in the calculation of his constant. Doubtless, by careful inspection, other useful facts may be elicited from this Table: one, however, of peculiar importance remains to be noticed, and the attention of the Academy is particularly invited to it.

Whereas we find calculated heights to be in excess during day-time, and in defect during night-time, there must be moments when the errors of such calculated heights are reduced to zero. Such are, obviously, the moments most favourable for barometric observations, and they are indicated by changes of sign from plus to minus, or from minus to plus.

By inspection of the large Table, I have ascertained the moments when the horary corrections vanish, which are given in the small Table which I here subjoin. Before I produce this small Table, I have to remark that the months of January and November have but one such moment; that December has *no* moment whatever when the horary correction = 0, all the calculated heights for this month being in defect, as indicated by the fact that all the horary

corrections of this month have the sign plus (+); that the remaining nine months, from February to October inclusive, have (each of them) two moments when the horary corrections vanish.

TABLE III.

Showing the Moments of the Forenoon and Afternoon, when the Horary Corrections vanish. Horary Correction = 0.

	A. M.	P. M.	
	H. M.	H. M.	
January, . .	. . .	1 0	January has but one moment when horary correction vanishes.
February, . .	10 0	4 0	
March, . . .	8 30	6 0	
April, . . .	7 20	7 0	
May, . . .	6 48	7 15	
June, . . .	6 30	8 0	
July, . . .	6 30	8 35	
August, . .	7 8	8 0	
September, .	8 15	6 20	
October, . .	9 20	4 27	
November, .	. . .	0 10	November, as January, has but one moment when horary correction vanishes, = 0.
December, . .	. . .	. . .	December has no moment that the horary correction vanishes, all the calculated heights for December being in defect.

The above small Table shows that from February to October inclusive, each month having two moments when the horary correction vanishes, such moments are nearly equally distant from 1 o'clock, P. M., when the errors in excess are greatest; also, that such moments are near to sunset, and to two hours from sunrise. It is, however, important to know that the time near sunset is more favourable for barometric observations than that of the forenoon. For, in examining the large Table of horary corrections, it appears that the corrections vary more rapidly in the forenoon than in the afternoon, near to the time when the horary corrections disappear. From these facts we learn that in making use of the new formula we require a Table of horary corrections, and that, when no such Table is to be had, we should be careful to make observations near to sunset, or to about two hours after sunrise.

Having thus proved by well-ascertained facts the serious errors to which barometric calculations are subject, even in employing the most approved formulæ, I have to remark that Tables of horary corrections, in the nature of things, have only a local application. Were proof of this required, we have it in the tract, " Sur le Nivellement du Grand-Saint-Bernard," at pages 11 and 12. M. PLANTAMOUR states that, having applied his small Table of horary corrections to two sets of barometric and hygrometric observations made by himself, in the valley which connects Martigny with the Convent of Saint Bernard, the use of his Table did good service as to one set of observations, but when applied to the other, actually introduced error equal in quantity to that of the horary correction.

It follows from this fact, that the large Table prepared by me is only good for the stations which have furnished the observations from which it has been calculated, namely, the Convent of the Great Saint Bernard and the Observatory of Geneva. *En attendant*, a sound local Table for the locality of Dublin and for other districts, I see nothing better to be done than to employ BAYLEY's formula for observations made near to noon of the summer months; and for other periods of the year to employ my own formula, or that of BESSEL, with the best Table of horary corrections which can be had; and when no table of corrections, trustworthy, is forthcoming, then to be careful to work with observations made near to sunset, or to two hours after sunrise. For, in employing BAYLEY's formula at such times, it virtually contains the horary correction. As to observations made near to sunset, or two hours after sunrise, my own formula contains implicitly the horary corrections, which vanish at such moments. I am now desirous to state to the Academy, in words as few as possible, the peculiar difference in form of the formula of BESSEL, and of my own, already published by the Academy.

Every one acquainted with barometric formulæ knows that such formula may be represented by $H = N \times \log \frac{B}{B'}$. I now speak of formulæ which make *no correct* allowance for the hygrometric state of the atmosphere, and, therefore, I make exception of the formula of BESSEL and of my own.

In this equation, viz., $H = N \times \log \frac{B}{B'}$, H indicates the height of one station

of observation above the other ; B and B' indicate the barometric pressures, fully and completely corrected; and N indicates the product of various factors relating to latitude, to mean temperature, &c., &c.

Now, if reference be made to my former paper, it will appear that my own formula, in making correct provision for the hygrometric state of the atmosphere, differs only from formulæ in previous use by replacing in such formula the expression $\log \frac{B}{B'}$ by the expression $\log \frac{B - \frac{3}{8}\sqrt{(ff')}}{B' - \frac{3}{8}\sqrt{(ff')}}$. Now, in reference to the formula of BESSEL, we find he replaces the expression $\log \frac{B}{B'}$ by the expression $\frac{1}{1 - \frac{\frac{3}{8}\sqrt{(ff')}}{\sqrt{BB'}}} \times \log \frac{B}{B'}$; so that if both these new formulæ be sound, we ought to have equality between the expressions,

$$\log \frac{B - \frac{3}{8}\sqrt{(ff')}}{B' - \frac{3}{8}\sqrt{(ff')}}, \text{ and } \frac{1}{1 - \frac{\frac{3}{8}\sqrt{(ff')}}{\sqrt{BB'}}} \times \log \frac{B}{B'}.$$

Now, let any one employ even extreme values of BB', and $\frac{3}{8}\sqrt{(ff')}$ in these expressions, and he will find the difference practically insensible ; for instance, in making use of the mean of the ten years' observations, hygrometric and barometric, made at the Convent of Saint Bernard and Observatory of Geneva, the heights calculated, according to BESSEL'S formula and my own, differ only by the hundreth part of a metre ; the heights by BESSEL being 2066·36 metres, and by my own formula being 2066·35 metres ; as to correctness, therefore, the one formula is as good as the other, but as to facility of calculation, mere inspection points out the superiority of the one over the other. There is, moreover, this peculiar difference between the two formulæ, that the differential equation, the integration of which has produced my formula (*vide* former paper) is *mathematically* correct, so that whenever the laws of the variation of atmospheric temperature and of the elastic forces of the vapour of water shall be ascertained exactly by experiment, we shall have, by a simple employment of the well-known rules of the integral calculus, a formula for barometric observations mathematically correct also. With respect to the formula of BESSEL, it is only a close approximation to truth, without being mathematically and strictly cor-

rect. This peculiarity of my own formula is but of little value if it give more trouble in actual calculations ; but as my formula is more simple in its working, its mathematical correctness is an additional recommendation of it.

I have to observe that the expression $\frac{3}{8}\sqrt{(ff')}$, which may be designated the hygrometric element, and which I have expressed, for sake of simplicity, by the character δ in the formulæ which I have given in the last pages of this paper, differs (but insensibly) from the corresponding hygrometric element of BESSEL's formula, which is $\frac{25610}{67407}aF_1$. In the last pages of this paper I employ the character δ' (delta aspirated), for sake of simplicity, to designate the quantity $\frac{25610}{67407}aF_1$; and in the Appendix I prove that for practical purposes $\delta=\delta'$, and that the one may, therefore, be employed indifferently for the other.

I have also given in the Appendix BESSEL's equation for calculating the elastic forces of vapour of water for the temperatures corresponding to such forces, and I have found such equation of BESSEL to give results very nearly the same as those I have obtained from the Tables of Dr. ANDERSON, calculated from experiments of DALTON and URE; the agreement is shown in the Appendix.

As the purport of this paper is not to explain the peculiarities of BESSEL's formula, I have to state that any one anxious to see it in full may do so by consulting M. PLANTAMOUR's quarto, "Resumé," at pp. 63, 64, 65, 66, and 67. M. PLANTAMOUR found it so obscure and different from other formulæ in general use, that he has devoted several pages in transforming it, and I frankly confess that, even thus transformed, it appears to me still very complicated. I have also taken the trouble to give it a form which may be found in the last pages of this Paper, and such form, I hope, will appear to the Academy sufficiently simple for the actual work of calculation. Nevertheless, though thus simplified, I can confidently pronounce it inferior, as to facility of calculation, to my own, which requires only, in addition to the labour of the old formulæ, the simple subtraction of the hygrometric element, $\delta=\frac{3}{8}\sqrt{(ff')}$ from the corrected barometric pressures.

Having now enlarged on the importance, and, I ought to say, the indispensable necessity of a sound Table of local horary corrections, in order to secure the

full benefit of the new formulæ, I have now to make known to the Academy, that, even with the help of such Table and new formulæ, I have reason to believe that the method of calculating heights by means of the barometer, hygrometer, and thermometer, is still subject to serious error.

A scientific periodical published at Geneva, under the direction of M. E. PLANTAMOUR, containing observations hygrometric, barometric, and thermometric, made at the Convent of the Great Saint Bernard and Observatory of Geneva, for the month of September of the year 1855, for every second hour between six hours A. M., and ten hours P. M., both inclusive, fell accidentally under my inspection, and I lost no time in making calculations with my own formula for the height of the Convent above the Observatory for such moments of the forenoon and afternoon as, according to the large Table of horary corrections, give the horary corrections $= 0$. Having done this, I was sadly disappointed at finding the calculated height to be nearly 12 metres different from the true height.

I wrote immediately to M. A. Morlott, residing at the time at Lausanne, requesting him to send me the observations for every month of the year 1855, extracted from the same scientific periodical; M. Morlott lost no time in kindly meeting my wishes.

With the observations for every month (as well as that of September) of the year 1855, I made calculations for the height of the Convent above the Observatory, at the moments when, according to large Table of horary corrections, the corrections vanish. The mean results of such calculations give very nearly the same error as the calculations for the month of September, viz., twelve metres. But, though disappointed, I did not feel any, the least, distrust in the new formula, being aware that the discrepancy can be explained by the mistake of assuming the arithmetic mean of temperatures as the true mean, which may sometimes be the case, but more often is not so. With respect to such mistake, the new formula is in no way responsible for it. That which the new formula proposed to do, it has done,—namely, to introduce a correct allowance for hygrometric action, with a correct constant. Of three distinct defects of the old formula two have been removed; one still remains, and to obviate its unhappy workings, we must look to Tables of horary corrections. The results above alluded to are given in the following Table:—

TABLE IV.

SHOWING *the Errors of Heights of Convent of Saint Bernard above Observatory of Geneva, calculated from Observations made when, according to large Table of Horary Correction, the Horary Corrections vanish. The calculated Heights are less than true Heights* (= 2070·34 *Metres*).

	A. M.	Errors.	P. M.	Errors.	
	H. M.	Metres.	H. M.	Metres.	
January.	. . .	. . .	1 0	1·76	One moment only.
February, ...	10 0	14·04	4 0	13·44	
March, ...	8 30	7·24	6 0	8·91	
April, ...	7 20	4·21	7 0	7·09	
May, ...	6 48	10·00	7 15	14·82	
June, ...	6 30	14·34	8 0	13·26	
July, ...	6 30	16·63	8 35	12·53	
August, ...	7 8	10·97	8 0	9·05	
September, ...	8 15	11·62	6 20	11·05	
October, ...	9 20	19·91	4 27	18·55	
November, ...	. . .	. . .	0 10	17·84	One moment only.
December, ...	. . .	. . .	. . .	. . .	No moment whatever.
Mean of nine months, February to October.	. . .	12·106	. . .	12·100	Mean of nine months, February to October.

Every person who may inspect the foregoing small Table will doubtless be surprised at perceiving, that although the mean of errors for the nine months between February and October (both inclusive), for the forenoon and afternoon, are very nearly equal (the one being 12·106 metres, the other being 12·100 metres), yet the errors for different months are strikingly different: for instance, the error of the month April of the forenoon is only 4·21 metres, but the corresponding error of the month October is as much as 19·91 metres. However, the errors of forenoon and afternoon are for the same month nearly equal. Now it is to be remarked, that according to the Table of Horary Corrections, I had reason to expect such errors to be little removed from nothing. Great, therefore, was my disappointment. Yet there is this consolation in such disappointment, that had the formula of LAPLACE, or any other derived from LAPLACE's (such as BAYLEY's or POISSON's), been employed instead of my own formula, the errors had been double of the actual errors, as given in the small

Table IV., so that although the Table of Horary Corrections meet not our wishes, nor even our expectations, it diminishes (by one half) the errors of the other formulæ. Therefore, let us not despond,—all we desire has not been realized, but considerable improvement has been made, and by diligence and zeal more may hereafter be done. The defect of the formula which now remains to be remedied is brought within small compass, and is simply the error occasioned by assuming an incorrect value of the real mean temperature of the atmospheric column between the stations of observations. Time only can tell if any considerable improvement be possible in reference to mean temperature errors, but these, if not removable by positive improvements in the formula itself, may be rendered for practical purposes comparatively innoxious, by sound tables of local horary correction; and in the formation of such tables the new formula, with new constants, will give, I believe, very important assistance.

I have now to bring forward the new formulæ contrasted with the old ones, as applied to the joint labours of M. A. Morlott and myself, near Montreux, the same appearing in four small Tables:—

Table V. shows the dates of observations, the calculated heights according to four principal formulæ, and the true height by actual spirit-levellings; these heights in Table V. have not as yet received the horary corrections.

Table VI. gives the calculated heights after the application of horary correction, as given by my large Table.

Table VII. gives the errors of heights as calculated by the four formulæ, *after* the application of horary corrections. And—

Table VIII. gives the influence of vapour of water of the atmosphere in increasing the height, such as may be obtained by employing a formula suited to a hypothetic atmosphere of dry air.

As the four heights, for which there are five observations (one of them being determined by a double set of observations) are unknown to the Academy, I shall designate them by the characters A, B, C, and D.

Table V.

Showing *the True Heights by Spirit-level; also the Calculated Heights, before the application of Horary Corrections.*

	Months.	P. M.	Bessel.	Renny.	Laplace.	Bayley.	True Height, Spirit-level.
	1857.	H. M.	English Feet.	English Feet.	English Feet.	English Feet.	English Feet.
A	April 29,	1 15	143·1	143·3	142·6	142·6	136·0
A	,,	6 30	136·3	136·4	135·7	135·7	136·0
B	,,	4 0	891·6	891·8	887·6	888·0	879·5
C	,,	4 0	1029·5	1029·5	1024·6	1025·0	1015·5
D	May 2, .	3 30	912 9	912·9	909·1	909·4	901·0

Table VI.

Showing *Heights after receiving Horary Corrections.*

	Months.	P. M.	Bessel.	Renny.	True Height, Spirit-level.
	1857.	H. M.	English Feet.	English Feet.	English Feet.
A	April 29, .	1 15	141·3	141·5	136·0
A	,,	6 30	136·1	136·2	136·0
B	,,	4 0	884·6	884·8	879·5
C	,,	4 0	1021·3	1021·3	1015·5
D	May 2, . .	3 30	904·7	904·7	901·0

Table VII.

Showing *the Errors of Calculated Heights, after receiving Horary Corrections.*

	Months.	P. M.	Bessel.	Renny.	Laplace.	Bayley.	True Height, Spirit level.
	1857.	H. M.	English Feet.	English Feet.	English Feet.	English Feet.	English Feet.
A	April 29,	1 15	5·3	5·5	6·6	6·6	136·0
A	,,	6 30	0·1	0·2	0·3	0·3	136·0
B	,,	4 0	5·1	5·3	8·1	8·5	879·5
C	,,	4 0	5·8	5·8	9·1	9·5	1015·5
C	May 2, .	3 30	3·7	3·7	8·1	8·4	901·0

TABLE VIII.

SHOWING *the Increase of Height due to the Influence of the Vapour of Water of the Atmosphere, such Increase being the Difference of Calculations made upon Supposition of the Atmosphere consisting of simple Dry Air, and of the Atmosphere being (as it is) impregnated with Vapour of Water.*

	Increase of Height by influence of Vapour of Water.				True Height by Spirit-levelling.
	Bessel.	Renny.	Laplace.	Bayley.	
	English Feet.	English Feet.	English Feet.	English Feet	English Feet.
A	0·45	0·46	0·34	0·34	136·0
B	2·80	2·82	1·68	1·68	879·5
C	3·60	3·60	1·90	1·90	1015·5
D	3·40	3·40	2·21	2·21	901·0

I have in the first place to remark, that according to Table VII., the error of height, A, is 5·5 feet by one observation, and only 0·2 feet by the second observation for same height. As the true height of A is only 136·0 feet, the error of the first observation cannot possibly be attributed to the incorrectness of the horary correction; therefore, it must be considered error of observation; and in reference to the heights B, C, and D of the same Table, the errors are nearly the same as the error of A; therefore, the errors of B, C, D are not beyond the possible errors of observation. But forasmuch as every calculated height near to Montreux is greater than the true heights by spirit-levelling, I am persuaded that the corrections applied to such calculated heights are too small, and this was to be expected from the locality of Montreux, a village renowned for the mildness and warmth of its climate.

I have in the second place to remark, that the errors connected with the formulæ of LAPLACE and of BAYLEY, as shown in Table VII., are obviously greater than the unavoidable errors of observation.

I have next to request the particular attention of the Academy to a proposal, on my part, of a formula having a peculiar constant, which shall dispense with hygrometric observations and hygrometric calculations on ordinary occasions; and yet shall make provision sufficiently correct for the hygrometric state of the atmosphere.

Before I do so, I have to correct a mistake made by me in my former paper,

relative to the recommendation of Poisson to increase the constant of Laplace by 57 metres, in order to remove from the right-hand side of the formula the unknown quantity z, being the required height of one station of observation above the other. The mistake to which I allude is this: I expressed my belief that the recommendation of Poisson was not good, because it occasioned errors greater than the unavoidable errors of observations,—in extreme cases it does so, but more often it does not. I now believe Poisson's advice to be good, for I have ascertained that in omitting the unknown quantity (or height) z, from the right-hand side of the formula applied to the height of the Convent of Saint Bernard above the Observatory of Geneva, an error of 6·17 metres in defect is occasioned, and that by increasing the constant of the formula by 57 metres, according to Poisson's recommendation, there is an increase of height to the amount of 6·39 metres,—the difference of 6·39 and 6·17 (being only 0·22 metres) is the total error caused by Poisson's recommendation. This error of 0·22 metres (less than 9·0 English inches), in a height of 2070·34 metres, is very much less than the errors of observation.

Moreover, in reference to the height C, viz., 1015·5 English feet, near Montreux, the error caused by Poisson's recommendation is only 0·36 English feet, being much less than the unavoidable error of observation. In general, therefore, Poisson's recommendation is excellent, seeing that it very much simplifies the calculations for height by the barometer. But the recommendation is not to be followed when the lower station is very much elevated above the level of the sea, such as the height of the Convent of Saint Bernard compared with the height of Mont Blanc. In such instances, when the height of the lower station (as the convent) is upwards of 8000 English feet above the level of the sea, the recommendation of Poisson is not applicable. But in the British Isles, and even in most parts of Switzerland, the recommendation should be followed. I purpose, in the formula which I am now about to propose, to combine Poisson's useful recommendation with my own contrivance, thereby producing a formula extremely simple, and which makes correct allowance for the hygrometric state of the atmosphere on ordinary occasions. My method is as follows :—

I calculate a constant from the equation $C' = \dfrac{0·76}{M \times D'}$ metres, in which D'

represents the ratio of the specific gravity, not of dry air, but of an union of dry air with vapour of water, having the elastic force belonging to the freezing-point, to that of quicksilver, in a manner analogous to that of obtaining the constant, 18404·9 metres of my principal formula, from the equation $C=\frac{0{\cdot}76}{M\times D}$ metres. The new constant $C'=18451{\cdot}5$ metres. The manner of calculating it is given in the Appendix, pp. 559 and 660. Now, following POISSON's recommendation of adding 57 metres, in order to remove z from the right-hand side of the formula, with a view to simplicity, we have, for the new value of C', 18508·5 metres. But this last constant, viz., 18508·5 metres, supposes the air to be in a state of saturation with vapour of water, which is seldom if ever the case. We have, therefore, to make a suitable deduction in consequence of the fraction of saturation, generally (if not always) accompanying the barometric observations. In order to do so, let us refer to Table I. of page 630, where we find that the increase in the calculated height of the Convent of Saint Bernard above the Observatory of Geneva, due to vapour of water in the atmosphere, amounts to 6·00 metres, very nearly. But it appears, in referring to M. PLANTAMOUR's quarto, entitled "Resumé," &c., &c., at page 51, that the mean fraction of saturation for heights, calculated in Table I. of page 630 of this paper, is 0·8; if, therefore, the decimal fraction 0·8 cause an increase of height = 6·00 metres, we should have an increase of 7·5 metres, if the atmosphere were in a state of saturation represented by unity, the difference between 7·5 and 6·0 being 1·5 metres, shows the diminution to be made on total height, in consequence of the fraction of saturation; but 1·5 being compared with the total height, 2066·35, gives a fraction $\frac{1}{1380}$ for suitable reduction of the constant last obtained, in order to obtain correct allowance for fraction of saturation. The same kind of reasoning being employed in reference to my observations near Montreux, gives for diminution of constant the fraction $\frac{1}{500}$ nearly, being greater than the first fraction $\frac{1}{1380}$, because of the greater temperature of observations near to Montreux. I propose, as a medium between these two values, the fraction $\frac{1}{1000}$, which is simple, and easily applied; therefore, taking the one-thousandth part of 18508·5 metres, viz., 18·5 metres; and subtracting the same, we have for the final value of constant $C'=18490{\cdot}0$ metres $=60664{\cdot}0$ English feet, very nearly.

I will now give the proposed modified formula, with its peculiar constant.

Let z be the required height of one station of observation above the other.

Let ϕ be the mean latitude of the stations of observation.

Let τ, τ' be temperatures (Centigrade) of atmosphere given by detached thermometer.

Let t, t' be temperatures (Fahrenheit) of atmosphere given by detached thermometer.

Let T, T' be temperatures (Centigrade) of quicksilver given by attached thermometer.

Let $\mathfrak{T}$, $\mathfrak{T}'$ be temperatures (Fahrenheit) of quicksilver given by attached thermometer.

Let β, β' be observed barometric pressures, *not* corrected for anything.

Let H. C. indicate horary corrections.

Centigrade Formula.

$$z = \overset{\text{Metres.}}{18490{\cdot}0}\ \{1 + 0{\cdot}002695\,.\cos 2\phi\} \left\{ \begin{array}{c} 1 + 0{\cdot}004\ \dfrac{\tau+\tau'}{2} \\ \text{and} \\ 1 + 0{\cdot}0035\ \dfrac{\tau+\tau'}{2} \end{array} \right\} \times$$

$$\times \log \frac{\beta}{\beta'.\{1+0{\cdot}00018.(T-T')\}} \pm (\text{H. C.})$$

Fahrenheit Formula.

$$z = \overset{\text{Eng. ft.}}{60664{\cdot}0}\ \{1 + 0{\cdot}002695\,.\cos 2\phi\} \left\{ \begin{array}{c} 1 + 0{\cdot}002222\left(\dfrac{t+t'}{2} - 32\right) \\ \text{and} \\ 1 + 0{\cdot}002\left(\dfrac{t+t'}{2} - 32\right) \end{array} \right\} \times$$

$$\times \log \frac{\beta}{\beta'.\{1+0{\cdot}0001.(\mathfrak{T}-\mathfrak{T}')\}} \pm (\text{H.C.}).$$

The multipliers 0·0035 and 0·002 must be employed in place of 0·004 and 0·0022222, &c., whenever the mean temperature of atmosphere, as given by the detached thermometers, may be below the freezing-point; but I have to remark,

that the decimal 0·002 is a very little greater than the value of Fahrenheit, which corresponds to 0·0035 of the Centigrade thermometer; such value is 0·0019444, &c., but by employing in place of this last value (for sake of simplicity) the fraction 0·002, no *sensible* error is occasioned. I have also to remark, that I have retained the increased value 0·004 instead of 0·00375 for expansion of air, according to LAPLACE's recommendation for temperatures above the freezing-point, having modified it for temperatures below it.

This formula is peculiarly simple; let us now test its accuracy by comparing its results with those of the complete formula already submitted to the Academy, and published in its Transactions.

TABLE IX.

SHOWING *the Results of Formula (now for the first time brought before the notice of the Academy), compared with those of the Correct Formula already before the Academy).*

Let, as before, H indicate the height of Convent Saint Bernard above Observatory of Geneva (*Vide*, page 630 of this paper).

Let, also, the heights near Montreux, as before, be designated A, B, C, and D (*vide* page 641 of this Paper).

Heights.	Renny's complete Formula, already before the Academy.	Renny's modified Formula, now for first time submitted to the Academy.	Errors of modified Formula.
	Metres.	Metres.	Metres.
H	2066·35	2066·08	0·27
	English Feet.	English Feet.	English Feet.
A	143·3	143·4	0·1
A	136·4	136·7	0·3
B	891·8	892·7	0·9
C	10295	1030·3	0·8
D	912·9	914·2	1·3

The column of errors of this last small Table gives results very gratifying indeed: the maximum error belonging to height, D, is still very much less than the unavoidable errors of observation.

It is, therefore, with confidence in its correct working, that I recommend this modified formula to the favourable consideration of the Academy.

I have, however, to protest against the employment of it, or of the more correct one already published, without the aid and co-operation of a reliable table of horary corrections, excepting at moments of the day near to sunset, or two hours after sunrise, when the horary corrections, if they do not vanish altogether, are comparatively but small.

In reference to BAYLEY's formula (as simple in form as that which I have now just described), I have to remark, that in the absence of a correct table of horary corrections, I advise the employment of it for observations made near noon of the summer months; because under such circumstances it contains implicitly the horary corrections. It is, in fact, the formula of POISSON, suited to the Fahrenheit thermometer and English measure, which has been obtained from LAPLACE's formula, of which the constant has been determined, as already stated, from observations of RAMOND, made in the Pyrenees near noon of the summer months.

I now produce BAYLEY's formula in juxtaposition with my own modified formula, for sake of comparison.

BAYLEY's *Formula—Fahrenheit Thermometer.*

$$z = 60345{\cdot}6 \text{ Eng. ft.} \{1 + 0{\cdot}002695 \cos 2\phi\} \left\{1 + 0{\cdot}002222\left(\frac{t+t'}{2} - 32\right)\right\} \times \log \frac{\beta}{\beta' \{1 + 0{\cdot}0001 (\mathfrak{T} - \mathfrak{T}')\}}.$$

The following is my own modified formula, which makes correct provision for the hygrometric state of the atmosphere, also for temperatures below the freezing-point, in which respect the formula of BAYLEY is *seriously* faulty:—

$$z = 60664{\cdot}0 \text{ Eng. ft.} \{1 + 0{\cdot}002695 \cos 2\phi\} \left\{\begin{array}{c} 1 + 0{\cdot}0022222\left(\frac{t+t'}{2} - 32\right) \\ \text{and} \\ 1 + 0{\cdot}002\left(\frac{t+t'}{2} - 32\right) \end{array}\right\} \times \log \frac{\beta}{\beta'(1 + 0{\cdot}0001 (\mathfrak{T} - \mathfrak{T}')} \pm (\text{H. C.}).$$

I have retained in my formula the values of the coefficients of the arithmetic mean of temperatures, given by detached thermometers, as recommended by LAPLACE, for temperatures above the freezing-point; but, as a matter of course, I have added a diminished multiplier for temperatures below the freezing-point.

In conclusion, I hope I may say, with confidence, and without presumption, that the questions of the new constant, and of the action of the vapour of water as affecting barometric calculations, need no further consideration. Of course, if hereafter the ratio of the specific gravity of dry air to that of quicksilver be more accurately determined by experiment than at present, the constant of my correct formula will be thereby improved; and, if also a more correct expression of the expansion of air, under increase of temperature, a corresponding improvement will thereby be obtained; but small indeed will be such improvements compared to those which will result from the employment of correct tables of horary correction, or from a more correct method than the present one of estimating the mean temperature of the atmospheric column between the stations of barometric observations. To these particular objects all attention ought now to be directed, for here at present is the weak side of barometric levelling. To obtain these objects, too many hands cannot be employed in making observations,—hygrometric, barometric, and thermometric,—the results of which, being compared with heights accurately determined by spirit-levelling, may furnish data for such purpose.

But such will require time; when, however, a considerable number of correct local tables of horary correction shall be produced, by a careful comparison of the same, it is not improbable that general tables may be formed, which shall be applicable to extensive portions of the Earth's surface, and by happy combinations of the logarithms of horary corrections with the logarithms of the other coefficients of the formula, methods of calculation may be contrived, which will give, with little trouble, beyond that of inspection, approximate and closely approximate values of the height of stations of barometric observations one above the other.

Having now fairly exhausted my subject, I will only add a slight summary of what I have already stated in detail.

The mode adopted by Laplace of making allowance for the hygrometric

state of the atmosphere, confessedly erroneous as to temperatures below the freezing-point, has been shown to be erroneous for temperatures also above the freezing-point. The constant recommended by LAPLACE has been proved to be too small, and has been replaced by a constant obtained by the consideration of the ratio of the specific gravity of dry air to that of quicksilver, and the constant thus obtained is necessarily more correct than a constant obtained by means of observations, however numerous, barometric, hygrometric, and thermometric. This better way of obtaining the constant has been rendered available by the peculiarities of the new formula, and it is in this point of view that the new formula is so peculiarly valuable.

The indispensable necessity of tables of horary correction, in order to obviate the mean temperature errors, is placed beyond the possibility of controversy, and the importance of the new formula as a valuable help to work out hereafter such tables has been made apparent.

But we are warned by the facts of this paper, that with even the assistance of a sound table of local horary corrections, we are not to expect exemption from serious error on all occasions ; it is, however, satisfactory to know that where error be not altogether excluded, it is diminished at least one-half by the working of the new system.

Moreover, the heights calculated by the new formula agree in a very satisfactory manner with the working of a formula of the same kind given by BESSEL, in use on the Continent. Its superiority to that of BESSEL appears by simple inspection. But forasmuch as the trouble of making hygrometric observations is not small, and as the advantage of such trouble on ordinary occasions is not great, in order to dispense with such trouble I have given a peculiar formula, peculiarly simple, which on ordinary occasions renders hygrometric observations unnecessary, and yet makes provision sufficiently correct for the hygrometric state of the atmosphere.

TABLE X., *showing the Mean Temperatures (Centigrade Thermometer) and Corresponding Horary Corrections during ten years, from the year*

(τ) indicates Temperature of Air at Lower Station; (τ') that of Upper Station. The temperatures are

Hours.	JANUARY.		FEBRUARY.		MARCH.		APRIL.		MAY.		JUNE.	
	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.
*0	− 3·99	+ 0·00043	− 1·19	− 0·00410	+ 1·58	− 0·00974	+ 5·56	− 0·01289	+ 10·44	− 0·01377	+ 13·69	− 0·01415
1	− 3·76	0·00000	− 0·85	− 0·00465	+ 1·88	− 0·00997	+ 5·82	− 0·01290	+ 10·71	− 0·01351	+ 14·04	− 0·01432
2	− 3·78	+ 0·00041	− 0·83	− 0·00401	+ 1·84	− 0·00896	+ 5·78	− 0·01160	+ 10·69	− 0·01236	+ 14·16	− 0·01377
3	− 4·00	+ 0·00142	− 1·12	− 0·00253	+ 1·51	− 0·00702	+ 5·48	− 0·00977	+ 10·42	− 0·01068	+ 14·05	− 0·01264
4	− 4·38	+ 0·00275	− 1·61	− 0·00050	+ 0·98	− 0·00462	+ 4·97	− 0·00720	+ 9·92	− 0·00858	+ 13·70	− 0·01076
5	− 4·80	+ 0·00468	− 2·23	+ 0·00179	+ 0·32	− 0·00200	+ 4·33	− 0·00469	+ 9·26	− 0·00578	+ 13·10	− 0·00832
6	− 5·17	+ 0·00607	− 2·79	+ 0·00369	− 0·33	+ 0·00034	+ 3·62	− 0·00221	+ 8·50	− 0·00327	+ 12·36	− 0·00558
7	− 5·46	+ 0·00697	− 3·28	+ 0·00530	− 0·93	+ 0·00240	+ 2·91	− 0·00009	+ 7·71	− 0·00065	+ 11·52	− 0·00281
8	− 5·64	+ 0·00759	− 3·65	+ 0·00633	− 1·46	+ 0·00407	+ 2·25	+ 0·00168	+ 6·94	+ 0·00166	+ 10·67	− 0·00026
9	− 5·75	+ 0·00784	− 3·88	+ 0·00708	− 1·90	+ 0·00533	+ 1·65	+ 0·00316	+ 6·22	+ 0·00363	+ 9·85	+ 0·00210
10	− 5·86	+ 0·00812	− 4·04	+ 0·00764	− 2·31	+ 0·00643	+ 1·10	+ 0·00441	+ 5·55	+ 0·00551	+ 9·08	+ 0·00422
11	− 5·96	+ 0·00834	− 4·16	+ 0·00822	− 2·71	+ 0·00756	+ 0·59	+ 0·00570	+ 4·89	+ 0·00716	+ 8·37	+ 0·00611
†12	− 6·10	+ 0·00875	− 4·28	+ 0·00871	− 3·15	+ 0·00889	+ 0·10	+ 0·00704	+ 4·27	+ 0·00868	+ 7·73	+ 0 00773
13	− 6·28	+ 0·00928	− 4·45	+ 0·00950	− 3·61	+ 0·01020	− 0·35	+ 0·00744	+ 3·69	+ 0·00994	+ 7·22	+ 0·00895
14	− 6·46	+ 0·00966	− 4·67	+ 0·01023	− 4·05	+ 0·01145	− 0·70	+ 0·00936	+ 3·25	+ 0·01064	+ 6·89	+ 0·00939
15	− 6·63	+ 0·01024	− 4·91	+ 0·01112	− 4·37	+ 0·01212	− 0·88	+ 0·00977	+ 3·05	+ 0·01059	+ 6·80	+ 0·00903
16	− 6·74	+ 0·01036	− 5·13	+ 0·01136	− 4·51	+ 0·01221	− 0·82	+ 0·00927	+ 3·17	+ 0·00935	+ 7·01	+ 0·00767
17	− 6·78	+ 0·01026	− 5·24	+ 0·01118	− 4·36	+ 0·01114	− 0·46	+ 0·00764	+ 3·68	+ 0·00682	+ 7·56	+ 0·00512
18	− 6·70	+ 0·00977	− 5·17	+ 0·01036	− 3·89	+ 0·00881	+ 0·20	+ 0·00488	+ 4·47	+ 0·00365	+ 8·38	+ 0·00189
19	− 6·47	+ 0·00874	− 4·86	+ 0·00841	− 3·12	+ 0·00558	+ 1·10	+ 0·00132	+ 5·63	− 0·00081	+ 9·38	− 0·00179
20	− 6·09	+ 0·00732	− 4·30	+ 0·00619	− 2·12	+ 0·00168	+ 2·16	− 0·00255	+ 6·81	− 0·00479	+ 10·44	− 0·00553
21	− 5·58	+ 0·00548	− 3·52	+ 0·00324	− 1·02	− 0·00234	+ 3·24	− 0·00653	+ 8·05	− 0·00841	+ 11·46	− 0·00879
22	− 4·99	+ 0·00346	− 2·65	+ 0·00024	+ 0·05	− 0·00582	+ 4·23	− 0·00971	+ 9·09	− 0·01139	+ 12·37	− 0·01130
23	− 4·43	+ 0·00199	− 1·82	− 0·00243	+ 0·95	− 0·00842	+ 5·03	− 0·01195	+ 9·90	− 0·01312	+ 13·12	− 0·01319
Mean,	− 5·49	+ 0·00624	− 3·36	+ 0·00468	− 1·45	+ 0·00205	+ 2·37	− 0·00085	+ 6·93	− 0·00123	+ 10·54	− 0·00254

* Noon. † Midnight.

obtained from a set of Observations made at the Convent of the Great Saint Bernard and the Geneva Observatory, 1841 *to the year* 1851, *both years inclusive.*

according to Centigrade Thermometer. $\frac{(\tau+\tau')}{2}$ indicates Mean Temperature of Air, Centigrade measure.

July.		August.		September.		October.		November.		December.		Hours.
$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	$\frac{(\tau+\tau')}{2}$	Horary Correction.	
+15·07	−0·01464	+14·15	−0·01240	+11·27	−0·00845	+6·48	−0·00455	+1·77	+0·00008	−2·02	+0·00435	0*
+15·43	−0·01482	+14·50	−0·01250	+11·60	−0·00858	+6·72	−0·00471	+2·01	−0·00038	−1·79	+0·00379	1
+15 57	−0·01420	+14·63	−0·01185	+11·69	−0·00787	+6·71	−0·00399	+1·96	+0·00042	−1·79	+0·00422	2
+15·44	−0·01282	+14·55	−0·01065	+11·55	−0·00648	+6·47	−0·00261	+1·69	+0·00169	−1·99	+0·00527	3
+15·12	−0·01109	+14·22	−0·00890	+11·21	−0·00470	+6·06	−0·00086	+1·26	+0·00333	−2·33	+0·00682	4
+14·54	−0·00864	+13·67	−0·00670	+10·70	−0·00265	+5·55	+0·00102	+0·78	+0·00499	−2·72	+0·00835	5
+13·84	−0·00613	+12·98	−0·00433	+10·09	−0·00060	+4·99	+0·00295	+0·32	+0·00635	−3·09	+0·00972	6
+13·03	−0·00360	+12·21	−0·00203	+ 9·45	+0·00129	+4·45	+0·00460	−0·05	+0·00741	−3·38	+0·01069	7
+12·19	−0·00112	+11·47	0·00000	+ 8·85	+0·00282	+3·96	+0·00613	−0·34	+0·00811	−3·57	+0·01112	8
+11·42	+0·00083	+10·78	+0·00168	+ 8·33	+0·00399	+3·54	+0·00737	−0·55	+0·00849	−3·68	+0·01125	9
+10·70	+0·00257	+10·14	+0·00320	+ 7·87	+0·00492	+3·17	+0·00831	−0·71	+0·00886	−3·74	+0·01132	10
+ 9·98	+0·00423	+ 9·52	+0·00477	+ 7·45	+0·00580	+2·85	+0·00945	−0·85	+0·00922	−3·76	+0·01123	11
+ 9·35	+0·00565	+ 8·88	+0·00642	+ 7·02	+0·00685	+2·57	+0·01026	−0·99	+0·00983	−3·79	+0·01123	12†
+ 8·74	+0·00711	+ 8·27	+0·00822	+ 6·55	+0·00818	+2·30	+0 01100	−1·14	+0·01046	−3·83	+0·01142	13
+ 8·29	+0·00809	+ 7·74	+0·00968	+ 6·10	+0·00949	+2·07	+0·01154	−1·31	+0·01094	−3·92	+0·01183	14
+ 8·11	+0·00809	+ 7·45	+0·01033	+ 5·74	+0·01042	+1·92	+0·01173	−1·48	+0·01154	−4·04	+0·01237	15
+ 8·21	+0·00720	+ 7·49	+0·00977	+ 5·60	+0·01062	+1 88	+0·01149	−1·62	+0·01186	−4·17	+0·01283	16
+ 8·71	+0·00500	+ 7·92	+0·00781	+ 5·75	+0·00967	+2·02	+0·01050	−1·68	+0·01179	−4·27	+0·01315	17
+ 9·55	+0·00183	+ 8·71	+0·00448	+ 6·25	+0·00760	+2·37	+0·00881	−1·58	+0·01130	−4·29	+0·01309	18
+10·57	−0·00200	+ 9·75	+0·00056	+ 7·03	+0·00446	+2·93	+0·00646	−1·28	+0·01006	−4·18	+0·01252	19
+11·72	−0·00593	+10·88	−0·00353	+ 7·99	+0·00083	+3·66	+0·00371	−0·80	+0·00822	−3·89	+0·01122	20
+12·80	−0·00930	+11·95	−0·00718	+ 9·00	−0·00260	+4·48	+0·00092	−0·13	+0·00585	−3·47	+0·00948	21
+13·74	−0·01189	+12·88	−0·00987	+ 9·94	−0·00544	+5·30	−0·00166	+0·60	+0·00343	−2·94	+0·00750	22
+14·53	−0·01381	+13·61	−0·01161	+10·71	−0·00750	+5·99	−0·00356	+1·27	+0·00150	−2·43	+0·00565	23
+11·94	−0·00331	+11·18	−0·00144	+ 8·66	+0·00134	+4·10	+0·00435	−0·12	+0·00689	−3·29	+0·00960	

* Noon. † Midnight.

Various Formulæ for determining the Height of Mountains, by means of Observations, Barometrical, Hygrometrical, and Thermometrical.

Let z be the height of one station above the other.

Let h be height above the level of the sea of lower station, supposed to be *known.*

Let h' be height above the level of the sea of upper station, as yet unknown.

Then $z=(h'-h)$.

Let r be radius of the Earth $=$ 6369668·0 metres $=$ 20898240·0 English feet.

Let $\frac{rh}{r+h}$ be A, a known quantity.

Let $\frac{rh'}{r+h'}$ be x, as yet unknown, to be determined by formula.

Now, it is easy to prove that $z=(h'-h)=(x-A)+\frac{x^2}{r}-\frac{A^2}{r}$, very nearly.

Let C be the constant for latitude 45°, at the level of the sea, at freezing-point, and $=$ 18404·9 metres $=$ 60384·6 English feet : *vide* Appendix.

Let ϕ be the mean latitude of stations of observation.

Let ζ be half the increase of gravity from the Equator to the Pole of the Earth $=0{\cdot}002695$.

Let k be expansion of air for one degree of Centigrade thermometer $=0{\cdot}003665$.

Let l be expansion of air for one degree of Fahrenheit $=0{\cdot}002036111$.

Let m be expansion of quicksilver for one degree Centigrade $=0{\cdot}00018$.

Let n be expansion of quicksilver for one degree Fahrenheit $=0{\cdot}0001$.

Let β, β' be barometric pressures of atmosphere at lower and upper stations of observation.

Let τ, τ' be temperatures (Centigrade) of the atmosphere at lower and upper stations of observation, as shown by the detached thermometers.

Let t, t' be temperatures (Fahrenheit) of the atmosphere at lower and upper stations of observation, as shown by the detached thermometers.

Let T, T' be the temperatures (Centigrade) of quicksilver of the barometers of the lower and upper stations, as shown by the attached thermometers.

Let $\mathfrak{T}$, $\mathfrak{T}'$ be the temperatures (Fahrenheit) of quicksilver of the barometers of the lower and upper stations, as shown by attached thermometers.

Let f, f' be the actual elastic forces of vapour of water of the atmosphere at the lower and upper stations, and very nearly equal to the elastic forces peculiar to the dew-points of the lower and upper stations, as given by an approved table of elastic forces (or tensions) of aqueous vapour, or otherwise obtained.

Let F_1 be the elastic force (or tension) of aqueous vapour, peculiar to the arithmetic mean of temperatures of the atmosphere given by the detached thermometers, as obtained from an approved table of elastic forces of vapour, or otherwise.

Let a be the mean fraction of saturation of the atmospheric column between the stations of observation, assumed to be equal to the arithmetic mean of fractions of saturation of atmosphere at lower and upper stations.

Let $\frac{3}{8}\sqrt{(ff')}$ be δ; δ being designated the hygrometric element.

Let $\frac{25610}{67407}\,aF_1$ be δ' ($\delta' = \delta$ very nearly: *vide* Appendix).

Let $\beta\left(1+\frac{2z}{r}\right)$ be B.

Let $\beta'\{1+m(T-T')\} = \beta'\{1+n(\mathfrak{T}-\mathfrak{T}')\}$ be B'.

B and B' are, therefore, the values of β, β' (barometric *observed* pressures) corrected for differences of the specific gravities of the quicksilver of the barometers, arising from differences of temperature of attached thermometers, as also from differences of gravity occasioned by height of one station above the other.

Let (H. C.) designate horary correction.

Renny's General Formula, in its most General Form.

$$z = C\{1+\zeta\cos 2\phi\}\left\{1+\frac{2h+z}{r}\right\}\left\{1+k\,\frac{\tau+\tau'}{2}\right\}\log\frac{B-\delta}{B'-\delta} \pm (\text{H. C.}), \text{ Cent.}$$

$$z = C\{1+\zeta\cos 2\phi\}\left\{1+\frac{2h+z}{r}\right\}\left\{1+l\left(\frac{t+t'}{2}-32\right)\right\}\log\frac{B-\delta}{B'-\delta} \pm (\text{H. C.}), \text{ Fahr.}$$

The character z in the above two formulæ, as also in all the following ones, replaces the character H as found in the formulæ of pages 444 and 446 of this volume; the character z is that employed by POISSON in his formula.

BESSEL'S *General Formula.*

$$(x-A)=C\{1+\zeta\cos 2\phi\}\left\{1+k\frac{\tau+\tau'}{2}\right\}\left\{\frac{1}{1-\frac{\delta'}{\sqrt{BB'}}}\right\}\log\frac{B}{B'}\pm(\text{H. C.}),\ \text{Cent.}$$

$$z=(x-A)+\frac{x^2}{r}-\frac{A^2}{r}.$$

RENNY'S *General Formula, having Constant and Coefficients in Figures.*

$$z=18404{\cdot}9\text{ metres}\{1+0{\cdot}002695\cos 2\phi\}\left\{1+\frac{2h+z}{r}\right\}\left\{1+0{\cdot}003665\frac{\tau+\tau'}{2}\right\}\times$$

$$\times\log\frac{B-\delta}{B'-\delta}\pm(\text{H. C.}),\ \text{Cent.}$$

$$z=60384{\cdot}6\text{ Eng. ft. }\{1+0{\cdot}002695\cos 2\phi\}\left\{1+\frac{2h+z}{r}\right\}\times$$

$$\times\left\{1+0{\cdot}00203611\left(\frac{t+t'}{2}-32\right)\right\}\log\frac{B-\delta}{B'-\delta}\pm(\text{H. C.}),\ \text{Fahr.}$$

BESSEL'S *General Formula, having Constant and Coefficients in Figures.*

$$(x-A)=\overset{\text{Metres.}}{18404{\cdot}8}\{1+0{\cdot}002695\cos 2\phi\}\left\{1+0{\cdot}003665\frac{\tau+\tau'}{2}\right\}\left\{\frac{1}{1-\frac{\delta'}{\sqrt{BB'}}}\right\}$$

$$\times\log\frac{B}{B'}\pm(\text{H. C.}).$$

$$z=(x-A)+\frac{x^2}{r}-\frac{A^2}{r}.$$

RENNY'S *Correct Formula, adapted to the Latitude of Dublin* $=53^\circ\ 23'$ N.

$$z=\overset{\text{Metres.}}{18390{\cdot}5}\left\{1+\frac{2h+z}{r}\right\}\left\{1+0{\cdot}003665\frac{\tau+\tau'}{2}\right\}\log\frac{B-\delta}{B'-\delta}\pm(\text{H. C,}).\ \text{Cent.}$$

$$z=\overset{\text{Eng. ft.}}{60337{\cdot}5}\left\{1+\frac{2h+z}{r}\right\}\left\{1+0{\cdot}002036111\left(\frac{t+t'}{2}-32\right)\right\}\log\frac{B-\delta}{B'-\delta}\pm(\text{H.C.}),\text{Fahr.}$$

RENNY'S *peculiar Formula, which dispenses with Hygrometric Observations; also removes* (z) *from the right-hand of the Equation.*

$$z = \overset{\text{Metres.}}{18490{\cdot}0}\,\{1 + 0{\cdot}002695 \cos 2\phi\} \left\{\begin{matrix} 1 + 0{\cdot}004\,\dfrac{\tau+\tau'}{2} \\ \text{and} \\ 1 + 0{\cdot}0035\,\dfrac{\tau+\tau'}{2} \end{matrix}\right\} \times$$

$$\times \log \frac{\beta'}{\beta'\,\{1 + 0{\cdot}00018(T - T')\}} \pm (\text{H. C.}), \text{ Cent.}$$

$$z = \overset{\text{Eng. ft}}{60664{\cdot}0}\,\{1 + 0{\cdot}002695 \cos 2\phi\} \left\{\begin{matrix} 1 + 0{\cdot}002222\left(\dfrac{t+t'}{2} - 32\right) \\ \text{and} \\ 1 + 0{\cdot}002\left(\dfrac{t+t'}{2} - 32\right) \end{matrix}\right\} \times$$

$$\times \log \frac{\beta}{\beta'\,\{1 + 0{\cdot}0001\,(\mathfrak{T} - \mathfrak{T}')\}} \pm (\text{II. C.}), \text{ Fahr.}$$

RENNY'S *peculiar Formula, which dispenses with Hygrometric Observations; also removes* (z), *the unknown quantity, from the right-hand side of the Formula, adapted to the Latitude of Dublin* = 53° 23′ N.

$$z = \overset{\text{Eng. ft.}}{60616{\cdot}7} \left\{\begin{matrix} 1 + 0{\cdot}0022222\left(\dfrac{t+t'}{2} - 32\right) \\ \text{and} \\ 1 + 0{\cdot}002\left(\dfrac{t+t'}{2} - 32\right) \end{matrix}\right\} \log \frac{\beta}{\beta'\,\{1 + 0{\cdot}0001\,(\mathfrak{T} - \mathfrak{T}')\}} \pm$$

$$\pm (\text{II. C.}), \text{ Fahr.}$$

BAYLEY'S *Formula, adapted to Latitude of Dublin* = 53° 23′.

$$z = \overset{\text{Eng. ft.}}{60298{\cdot}7}\,\{1 + 0{\cdot}0022222, \&\text{c.}\left(\frac{t+t'}{2} - 32\right) \log \frac{\beta}{\beta'\,\{1 + 0{\cdot}0001\,(\mathfrak{T} - \mathfrak{T}')\}}.$$

Mr. RENNY'S formula (when there are no tables of horary correction) ought only to be used near sunset, or two hours after sunrise. BAYLEY'S formula can be used with safety *only* for observations made near noon, during the fine weather of the summer season, and when temperature is above the freezing-point.

Table XI.—*Horary Corrections suited to the Convent of*

In order to make use of the Table of Horary Corrections, multiply the height obtained by calculation from the Barometric Observations, by the calculated height

Hours.	January.	February.	March.	April.	May.	June.
0*	+0·00043	−0·00410	−0·00974	−0·01289	−0·01377	−0·01415
1	0·00000	−0·00465	−0·00997	−0·01290	−0·01351	−0·01432
2	+0·00041	−0·00401	−0·00896	−0·01160	−0·01236	−0·01377
3	+0·00142	−0·00253	−0·00702	−0·00977	−0·01068	−0·01264
4	+0·00275	−0·00050	−0·00462	−0·00720	−0·00858	−0·01076
5	+0·00468	+0·00179	−0·00200	−0·00469	−0·00578	−0·00832
6	+0·00607	+0·00369	+0·00034	−0·00221	−0·00327	−0·00558
7	+0·00697	+0·00530	+0·00240	−0·00009	−0·00065	−0·00281
8	+0·00759	+0·00633	+0·00407	+0·00168	+0·00166	−0·00026
9	+0·00784	+0·00708	+0·00533	+0·00316	+0·00363	+0·00210
10	+0·00812	+0·00764	+0·00643	+0·00441	+0·00551	+0·00422
11	+0·00834	+0·00822	+0·00756	+0·00570	+0·00716	+0·00611
12†	+0·00875	+0·00871	+0·00889	+0·00704	+0·00868	+0·00773
13	+0·00928	+0·00950	+0·01020	+0·00744	+0·00994	+0·00895
14	+0·00966	+0·01023	+0·01145	+0·00936	+0·01064	+0·00939
15	+0·01024	+0·01112	+0·01212	+0·00977	+0·01059	+0·00903
16	+0·01036	+0·01136	+0·01221	+0·00927	+0·00935	+0·00767
17	+0·01026	+0·01118	+0·01114	+0·00764	+0·00682	+0·00512
18	+0·00977	+0·01036	+0·00881	+0·00488	+0·00365	+0·00189
19	+0·00874	+0·00841	+0·00558	+0·00132	−0·00081	−0·00179
20	+0·00732	+0·00619	+0·00168	−0·00255	−0·00479	−0·00553
21	+0·00548	+0·00324	−0·00234	−0·00653	−0·00841	−0·00879
22	+0·00346	+0·00024	−0·00582	−0·00971	−0·01139	−0·01130
23	+0·00199	−0·00243	−0·00842	−0·01195	−0·01312	−0·01319
Mean, . . .	+0·00624	+0·00468	+0·00205	−0·00085	−0·00123	−0·00254

* Noon. † Midnight.

the Great Saint Bernard and the Geneva Observatory.

Correction obtained from the Table for the hour and month of the year, then add or subtract the product (according to the sign) to or from the for the true height.

July.	August.	September.	October.	November.	December.	Hours.
−0·01464	−0·01240	−0·00845	−0·00455	+0·00008	+0·00435	12 Noon.
−0·01482	−0·01250	−0·00858	−0·00471	−0·00038	+0·00379	1 Afternoon, or P. M.
−0·01420	−0·01185	−0·00787	−0·00399	+0·00042	+0·00422	2
−0·01282	−0·01065	−0·00648	−0·00261	+0·00169	+0·00527	3
−0·01109	−0·00890	−0·00470	−0·00086	+0·00333	+0·00682	4
−0·00864	−0·00670	−0·00265	+0·00102	+0·00499	+0·00835	5
−0·00613	−0·00433	−0·00060	+0·00295	+0·00635	+0·00972	6
−0·00360	−0·00203	+0·00129	+0·00460	+0·00741	+0·01069	7
−0·00112	0·00000	+0·00282	+0·00613	+0·00811	+0·01112	8
+0·00083	+0·00168	+0·00399	+0·00737	+0·00849	+0·01125	9
+0·00257	+0·00320	+0·00492	+0·00831	+0·00886	+0·01132	10
+0·00423	+0·00477	+0·00580	+0·00945	+0·00922	+0·01123	11
+0·00565	+0·00642	+0·00685	+0·01026	+0·00983	+0·01123	12 Midnight
+0·00711	+0·00822	+0·00818	+0·01100	+0·01046	+0·01142	1 Morning, or A. M.
+0·00809	+0·00968	+0·00949	+0·01154	+0·01094	+0·01183	2
+0·00809	+0·01033	+0·01042	+0·01173	+0·01154	+0·01237	3
+0·00720	+0·00977	+0·01062	+0·01149	+0·01186	+0·01283	4
+0·00500	+0·00781	+0·00967	+0·01050	+0·01179	+0·01315	5
+0·00183	+0·00448	+0·00760	+0·00881	+0·01130	+0·01309	6
−0·00200	+0·00056	+0·00446	+0·00646	+0·01006	+0·01252	7
−0·00593	−0·00353	+0·00083	+0·00371	+0·00822	+0·01122	8
−0·00930	−0·00718	−0·00260	+0·00092	+0·00585	+0·00948	9
−0·01189	−0·00987	−0·00544	−0·00166	+0·00343	+0·00750	10
−0·01381	−0·01161	−0·00750	−0·00356	+0·00150	+0·00565	11
−0·00331	−0·00144	+0·00134	+0·00435	+0·00689	+0·00960	Mean Horary Corrections.

APPENDIX.

ON THE CONSTANTS OF THE BAROMETRIC FORMULÆ.

THE expression $C = \frac{0\cdot76}{M \times D}$ metres, by which is calculated the constant of a barometric formula for a suppositious atmosphere of simple dry air, perfectly free from vapour of water, in which expression C represents the constant of the formula, M the modulus of common logarithms, and D the ratio of the specific gravity of dry air to that of quicksilver, under a pressure of 0·76 metres of quicksilver, may be obtained in the following manner :—

It is obvious that the height of a column of dry air of uniform density $= \frac{0\cdot76}{D}$ metres. Let us now suppose a column of dry air to be divided into an extremely great number of strata of equal thickness. Let n be a number extremely great: consequently the thickness of the lowest stratum (as also that of every other stratum), or the distance between the lower and upper surfaces of such lowest stratum, may be represented by the quantity $\frac{1}{n}$ metre. It is also obvious that the density of such stratum (though not mathematically uniform) has uniformity for its limit of variableness. If we now assume as measure of the pressure at the lower surface of the lowest stratum, the height of a column of dry air of uniform density $= \frac{0\cdot76}{D}$ metres, we have for measure of pressure at the upper surface, distant from the lower surface, by a quantity $= \frac{1}{n}$ metre, the expression $\frac{0\cdot76}{D} - \frac{1}{n}$ metres. But n being a number extremely or indefinitely great, in place of these expressions we may employ the expressions $\frac{0\cdot76}{D} + \frac{1}{n}$ and $\frac{0\cdot76}{D}$; making use of the well-known barometric formula $\left(h = C \times \text{com. log. } \frac{p}{p'}\right)$ in which h is the height of one station above the other, C is a constant quantity, and p, p' are the pressures at the lower and upper stations, we have $\frac{1}{n} = C \times \text{com. log. } \left\{\frac{\frac{0\cdot76}{D} + \frac{1}{n}}{\frac{0\cdot76}{D}}\right\}$;

or,

$$\frac{1}{n} = C \times M \times \text{hyp. log. } \left\{1 + \frac{1}{n}\frac{D}{0\cdot76}\right\} = C \times M \left\{\frac{1}{n}\frac{D}{0\cdot76} - \frac{1}{2}\left(\frac{1}{n}\frac{D}{0\cdot76}\right)^2 + \tfrac{1}{3}\ \&c.\right\};$$

or,

$$C = \frac{1}{M\left\{\frac{D}{0\cdot 76} - \frac{1}{n}\frac{1}{2}\left(\frac{D}{0\cdot 76}\right)^2 + \frac{1}{n^2}\frac{1}{3}\left(\frac{D}{0\cdot 76}\right)^3 - \&c.\right\}}.$$

But D is a fixed quantity, very small, and n is a number indefinitely great; we may, therefore, omit in the denominator of the right-hand side of the last equation, every term after the first. Doing so, we have—

$$C = \frac{0\cdot 76}{M \times D} \text{ metres.}—Q. E. D.$$

Now $M = 0\cdot 43429$, and from calculations made by experiments by REGNAULT at Paris, we have at freezing-point, for the value of D at the level of the sea of latitude 45°, the expression $D = \frac{1}{10517\cdot 3}$; these values being substituted in the equation $C = \frac{0\cdot 76}{M \times D}$ metres, we have $C = 18404\cdot 9$ metres, consequently C (the constant for an atmosphere of dry air, at the freezing-point, at the level of the sea of latitude 45°) = 18404·9 metres.

Let us now calculate a constant C' for an atmosphere consisting of an union or mixture of dry air and vapour of water, having an elastic force (or tension) represented by F metres, peculiar to the freezing-point, such as may be found in an approved table of elastic forces of vapour of water, or obtained in any other manner correctly, such union or mixture being under a pressure of 0·76 metres of quicksilver.

Let D' be the ratio of the specific gravity of such an union or mixture of dry air and vapour of water (D being the corresponding ratio of dry air alone) to that of quicksilver, under a pressure of 0·76 metres of quicksilver at freezing-point.

Now it is well known that $D' = D\left\{1 - \frac{\frac{3}{8}F}{0\cdot 76}\right\}$, for proof of which, *vide* former paper by Mr. RENNY, Article 2, p. 439, *supra*.

But $\frac{3}{8}F = 0\cdot 00190$ metres, and $D = \frac{1}{10517\cdot 3}$, therefore we have—

$$D' = \frac{1}{10517\cdot 3}\left\{1 - \frac{0\cdot 00190}{0\cdot 76000}\right\} = \frac{1}{10543\cdot 7} \text{ metres.}$$

But $C' = \frac{0\cdot 76}{M \times D'}$ metres $= \frac{0\cdot 76}{0\cdot 43429 \times \frac{1}{10543\cdot 7}}$ metres $= 18451\cdot 5$ metres.

Reproducing our constants, we have—

$C = 18404\cdot 9$ metres $\left\{\begin{array}{l}\text{Being the constant of a barometric formula for a suppositious} \\ \text{atmosphere of dry air, at the level of the sea, freezing-point, of} \\ \text{latitude 45°.}\end{array}\right.$

$C' = 18451\cdot5$ metres $\left\{\right.$ Being the constant for an atmosphere (consisting of an union or mixture of dry air and vapour of water having an elastic force (or tension) = 0·00506 metres, peculiar to the freezing-point, as obtained from an approved Table of elastic forces of vapour of water, or otherwise correctly obtained) at level of the sea, of latitude 45°, at freezing-point.

In order to compare the formula of BESSEL (employed by M. E. PLANTAMOUR in calculating the height of the Convent of the Great Saint Bernard above the Observatory of Geneva) with my own formula already submitted to the Royal Irish Academy, it is desirable to prove that the expression ($\frac{3}{8}\sqrt{ff'}$, being three-eighths of the square root of the product of the elastic forces of vapour of water, corresponding to the dew-points) is (*quam proxime*) equal to $\frac{25610}{67407}\,aF_1$ of BESSEL'S formula, in which expression a represents the arithmetic mean of fractions of saturation at the stations of observation, and F_1 represents the elastic force (or tension) of vapour of water, corresponding to the arithmetic mean of temperatures of stations, as given by the detached thermometers, obtained from an approved table of elastic forces of vapour of water, or calculated by the equation of BESSEL (given in the Appendix, *vide* page 661), or otherwise correctly obtained.

In order to prove that (for practical purposes) $\frac{3}{8}\sqrt{ff'} = \frac{25610}{67407}\,aF_1$, and that it is a matter of indifference, in barometric formulæ, whether we make use of the one or the other, I have to remark that the elastic forces of vapour of water corresponding to the dew-points of the atmosphere, are very nearly equal to the elastic forces of the vapour of water of the atmosphere. Now if we consult any approved table of elastic forces of vapour of water (such as that of Dr. ANDERSON, calculated from experiments by DALTON and URE), we shall find that the elastic forces form a geometric series (*quam proxime*) when the corresponding temperatures form an arithmetic one, consequently $\sqrt{ff'}$ indicates the elastic force of vapour of water belonging to a stratum of the atmosphere, situated half-way between the upper and lower stations of observation. But as F_1 indicates the elastic force of vapour of water corresponding to the arithmetic mean of temperatures of stations, belonging to a stratum half-way between the stations of observation (obtained from an approved table of elastic forces of aqueous vapour, which supposes a state of *saturation*), it is obvious that to obtain the actual elastic force we must multiply F_1 by the fraction of saturation represented by a. Doing so, we have the expression aF_1. Therefore (*quam proxime*), $\sqrt{(ff')} = aF_1$. But, by trial, it is easily discovered that (*quam proxime*), we have—

$$\tfrac{3}{8} = \frac{25610}{67407}; \text{ therefore } \tfrac{3}{8}\sqrt{ff'} = \frac{25610}{67407}\,aF_1 \text{ (\textit{quam proxime}).}$$

Seeing that these expressions are very small in *quantity*, it is an affair of perfect indifference whether we employ the one or other, in barometric formulæ. In my paper on

the Constants of Barometric Formulæ, which accompanies this Appendix, I have represented the expression $\frac{3}{8}\sqrt{ff'}$ by the character δ, and the expression $\frac{25610}{67407}\,aF_1$, by the character δ'. So that for practical purposes, $\delta = \delta'$.—*Q. E. D.*

I now bring forward BESSEL'S equation for calculating the elastic forces of vapour of water (expressed in metres), corresponding to temperatures of Centigrade thermometer, accompanied by some constant logarithms, useful in facilitating calculation.

Let F_1 be the elastic force (or tension) of vapour of water, corresponding to temperature $\frac{\tau + \tau'}{2}$ (being the arithmetic mean of temperatures, as given by the detached thermometers).

$$F_1 = \overset{\text{Metres.}}{0{\cdot}0051229} \times 10^{\,+\,0{\cdot}0279712\,\frac{\tau+\tau'}{2}\,-\,0{\cdot}0000625826\left(\frac{\tau+\tau'}{9}\right)^2}.$$

$$\begin{aligned}
\text{Log } 0{\cdot}0051229 &= \bar{3}{\cdot}7095159.\\
\text{Log } 0{\cdot}0279712 &= \bar{2}{\cdot}4467111.\\
\text{Log } 0{\cdot}0000625826 &= \bar{5}{\cdot}7964536.
\end{aligned}$$

BESSEL'S *Equation for calculating Elastic Forces (or Tension) of Vapour of Water, adapted to English inches and Fahrenheit thermometer.*

Let F be the elastic force, corresponding to temperature t, Fahrenheit.

$$F = \overset{\text{Eng. In.}}{0{\cdot}0613265} \times 10^{\,+\,0{\cdot}0167757\,.\,t^1\,-\,0{\cdot}0000193156\,.\,t^2}.$$

$$\begin{aligned}
\text{Log } 0{\cdot}0613265 &= \bar{2}{\cdot}7876482.\\
\text{Log } 0{\cdot}0167757 &= \bar{2}{\cdot}2246806.\\
\text{Log } 0{\cdot}0000193156 &= \bar{5}{\cdot}2859082.
\end{aligned}$$

TABLE *of Elastic Forces (or Tensions) of Vapour of Water, corresponding to the Temperatures (Fahrenheit)* 0°, 32°, 50°, *and* 100°, *as calculated by the above Equation of* BESSEL, *and as obtained from* ANDERSON'S *Tables, made from experiments by* DALTON *and* URE.

Fahrenheit.	Bessel.	Anderson.	Differences.
Degrees.	Inches.	Inches.	Inches.
0	0·06133	0·06121	0·00012
32° Freezing-point.	0·20169	0·19934	0·00235
50°	0·37856	0·37345	0·00511
100°	1·87090	1·85241	0·01849

TABLE XII.

	Metres.	English Feet.
Height above level of the sea, by accurate spirit-levelling of the cuvette of the barometer of the Convent of the Great Saint Bernard, ...	2478·34	8131·18
Height above level of the sea, by accurate spirit-levelling of the cuvette of the barometer of the Observatory of Geneva, ...	408·00	1338·61
Height above level of the sea, by accurate spirit-levelling of the zero of the limnimetre, Geneva,	373·86	1226·60
Height above the level of the sea by accurate spirit-levelling of the mean level of the surface of Lake Geneva, ...	375·92	1233·35
Height of Mont Blanc above the cuvette of the barometer of the Convent of Great Saint Bernard according to barometric calculations, made from observations, hygrometric as well as barometric, made by Messrs. BRAVAIS and MARTINS, during their ascent of Mont Blanc, 29th August, 1844, compared with corresponding observations made at the Convent, ...	2339·0	7674·02
Height of Mont Blanc above the level of the sea, .	4817·34	15805·20

POSTSCRIPT.

(Read May 9, 1859.)

In the foregoing paper I particularly pointed out that, notwithstanding the great improvements produced by my new Formulæ, having new Constants, assisted by Tables of Local Horary Correction, we ought not to expect exemption from serious error on all future occasions. My reasons for this my belief were stated at pages 638, 639, and 640. At page 639 commences the following statement:—

"Now, it is to be remarked, that according to the Table of Horary Corrections, I had reason to expect such errors to be little removed from nothing. Great, therefore, was my disappointment. Yet there is this consolation in such disappointment, that had the formula of Laplace, or any other derived from Laplace (such as Bayley's or Poisson's) been employed instead of my own formula, the errors had been double of the actual errors, as given in the small Table IV., so that, although the Table of Horary Corrections meet not our wishes nor our expectations, it diminishes by one-half the errors of the other formulæ. Therefore, let us not despond ; all we desire has not been realized, but considerable improvement has been made, and by diligence and zeal more may hereafter be done. The defect of the formula which now remains to be remedied is brought within a small compass, and is simply the error occasioned by assuming an incorrect value of the mean temperature of the atmospheric column between the stations of observations. Time only can tell if any considerable improvement be possible in reference to mean-temperature errors, but these, if not removable by positive improvements in the formula itself, may be rendered for practical purposes comparatively innoxious, by sound tables of local horary correction ; and in the formation of such tables, the new formula with new constant, will give, I believe, very important assistance."

Now, in reference to this extract, I have to remark, that in consequence of information lately communicated to me by Monsieur E. Plantamour, I have to modify, very considerably, my opinion concerning the mean-temperature

errors of barometric formulæ; not that I doubt that such errors are still very serious, but I do not now believe them to be grievous to the extent I believed them to be, when I prepared the Table of Errors (Table IV., page 639 of this volume). For I am now aware that the said Table IV., page 639, has been prepared from incorrect information, published by Monsieur E. PLANTAMOUR in a scientific periodical, which appears in monthly parts at Geneva. It is not for me to point out to Monsieur E. PLANTAMOUR the mode in which such periodical should be conducted, but, in justice to myself, I have to state my regret that inaccurate information should have been thus given to the public, whereby I have been led into a serious mistake concerning the mean-temperature errors of barometric formulæ, and have been induced to occupy unnecessarily the time and attention of the Royal Irish Academy. It now appears that the barometers of the Convent of the Great Saint Bernard and of the Geneva Observatory have, each, a constant error,—the error of the barometer of the Convent being 0·70 millimetres in excess; that of the barometer of the Geneva Observatory being 0·79 millimetres in defect. It also appears, that in the scientific periodical published at Geneva, in monthly parts, under the direction of Monsieur E. PLANTAMOUR, the constant error of the barometer of the Convent has been applied to the published observations made at the Convent since the year 1845, so that such published barometric observations are correctly given; but, unhappily, similar precautions have not been practised in reference to the published observations of the Geneva Observatory, so that such observations require a correction to the amount of 0·79 millimetres. True it is, that once a year, near the middle part of the year, summer time, Monsieur E. PLANTAMOUR publishes in the said scientific periodical, above alluded to, a Resumé, in full detail, of observations, barometric, thermometric, hygrometric, &c., &c., &c., of the past year; but seeing that I had no possible opportunity of becoming acquainted with such Resumé, I have,u nfortunately, been employing incorrect data in working out my Table, No. IV., page 639. Perhaps it may occur to some Members of the Academy, that although it did not seem good to Monsieur E. PLANTAMOUR to apply the necessary correction (viz., $+0{\cdot}79^{\mathrm{mm}}$) to the barometric observations made at the Geneva Observatory, as he has done to those of the Convent Saint Bernard,—yet, surely, he has not omitted, in each monthly publication, an intimation of some kind or other, that

the correction ($+ 0{\cdot}79^{mm}$) is necessary, in order to bring the published barometric observations to the true and real value of the observations ; but nothing of the kind appears ; no, not even a slight statement in the pages of the periodical containing the observations, that they require correction. Therefore, in justice to myself, I am under the necessity of thus clearly stating the case, lest any Member of the Academy should consider me careless or negligent in this affair. Indeed, without the gift of something approaching to inspiration, it had not been possible for me to have acted in this matter otherwise than I have done. Having thus explained to the Academy how it has happened that I have been employed (unintentionally) in misleading the Academy,—as a matter of course, it is now necessary to dismiss, at once and for ever, the Table of Errors, No. IV. of page 639, in order to make way for a new Table of Errors calculated with correct data.

The following Table is intended to show the Errors of Height of the Convent of the Great Saint Bernard above the Geneva Observatory, calculated from correct data, for moments of the day, mean time, when, according to the large Table of Horary Corrections (*vide* Table III., page 634, Table X., page 650; also Table XI., page 656), the horary correction vanishes. This new Table contains also the moments, mean time, of sunrise and sunset, for the middle day of each month of the year, for the mean latitude of the Convent and Observatory (viz., 46° 2′ North). The correct data for the year 1855, by means of which the contents of this Table have been calculated, may be found in the "Archives des Sciences Physiques et Naturelles, Bibliotheque Universelle de Geneve," Juillet, 1856, tome 32, de la 4me serie, No. 127, under the Article "Resumé Metéorologique de l'année 1855, pour Genève et le Grand Saint Bernard," par M. le Professeur Plantamour, commencing at page 177 of the "Archives des Sciences Physiques et Naturelles," and terminating at page 203 of the same Archives, &c. The Archives here alluded to are to be found in the Library of the Royal Irish Academy. The true height of the Convent above the Observatory, by extremely accurate spirit-levelling = 2070·34 metres; the mean latitude of the Convent and Observatory = 46° 2′ North; horary correction = 0, according to Tables X. and XI.; *vide* pages 650 and 656; also Table III., page 634.

TABLE XIII.

1855. Months.	Sunrise, mean time. Middle day of each month.	A. M. Moments, mean time, when Horary Corrections vanish, according to Table III.	Errors of Calculated Heights.	Sunset, mean time. Middle day of each month.	P. M. Moments, mean time, when Horary Corrections vanish, according to Table III.	Errors of Calculated Heights.	Remarks.
	H. M.	H. M.	Metres.	H. M.	H. M.	Metres.	
January, ...	7 44	. . .	. . .	4 35	1 0	+ 4·62	One moment only.
February, ...	7 9	10 0	− 5·30	5 20	4 0	− 4·70	
March, ...	6 18	8 30	+ 1·32	6 0	6 0	− 2·05	
April, ...	5 20	7 20	+ 4·54	6 40	7 0	+ 1·53	
May, ...	4 34	6 48	− 1·28	7 19	7 15	− 3·44	
June, ...	4 14	6 30	− 5·40	7 46	8 0	− 3·62	
July, ...	4 29	6 30	− 7·48	7 42	8 35	− 3·48	
August, ...	5 3	7 8	− 2·00	7 4	8 0	+ 0·07	
September, ...	5 42	8 15	− 2·61	6 9	6 20	− 2·00	
October, ...	6 22	9 20	− 10·94	5 10	4 27	− 9·55	
November, ...	7 6	. . .	. . .	4 24	0 10	− 10·01	One moment only.
December, ...	7 41	. . .	. . .	4 9	. . .	. . .	No moment whatever.
Mean of nine months, from February to October, both inclusive.	H. M. 5 28	H. M. 7 49	Metres. − 3·24	H. M. 6 34	H. M. 6 37	Metres. − 3·03	Mean of nine months, from February to October, both inclusive.

Comparing the above Table XIII. with similar Table IV. of page 639, I rejoice to be able to modify my opinion very much indeed, concerning the merits of the large Tables of Horary Corrections of pages 650 and 657. The column of errors, P. M., of the above Table, furnishes facts very satisfactory indeed, particularly for the seven months from March to September P. M., both months inclusive. The mean error for these seven months P. M. is only 1·85 metres. Now, it is not to be forgotten, that the vanishing moments of Table III., page 634, were obtained as the mean of ten years' observation, from 1841 to 1850, both years inclusive, so that the deviations of one particular year (viz., 1855) from the mean of ten previous years, is small indeed. Moreover, M. PLANTAMOUR informs us, at page 184 of the "Resumé Meteorologique" for the year 1855, that the mean temperature for this year (1855) was one-tenth of a degree colder than the mean temperature of the twenty preceding years, so that, making allowance for this circumstance, the mean error of calculated heights, P.M., for the seven months from March to September, being only 1·85 metres, will be

reduced to 1·1 metre, being an error very small indeed, about one in two thousand parts very nearly.

With respect to errors of heights of the forenoon (from two hours to two hours and a half after sunrise), when the horary correction vanishes, it appears by inspection of Table XIII. that such errors are more variable and unsteady than the corresponding errors near sunset. For example, the error for the month of July is as much as 7·48 metres, that of June being 5·40 metres, and that of April 4·54 metres, although the mean error for the seven months of the forenoon, from March to September (both months inclusive), is only 1·84 metres, being but the one-hundredth of a metre different from the corresponding mean of the afternoon (viz., 1·85 metres).

These facts clearly indicate sunset as the moment of the day most favourable for barometric observations, and the comparison of mean sunsets for the nine months (which have two moments, that the horary corrections vanish), with the mean moments of adjoining column, shows the same thing; for by Table XIII., the mean of sunsets is 6 h. 34 m., being different only by three minutes of time from the mean of moments of observation of adjoining column, whereas the mean of sunrises differs about two hours and twenty minutes from the mean of adjoining column. In my paper on the Constants of the Barometric Formulæ, to which this, my present paper, is but a supplement, I have recommended (in absence of a reliable table of horary correction) sunset and two hours after sunrise, as the moments of the day most favourable for barometric work. I now feel disposed to modify my opinion, as to observations of the forenoon, and to recommend, as the best moment, the moment equally distant with sunset, from the moment of highest temperature of the day, generally between one and two hours, P. M.

As to other periods of the day, I should not be afraid to make calculations for height by my own formulæ (in absence of a sound local table of corrections), making use of the large Table X. of page 650, for the seven months from March to September (both months included), particularly if the arithmetic mean temperatures, as given by the detached thermometers, be not widely different from the corresponding arithmetic means of temperature of Table X. I do not, however, feel myself authorized to recommend such practice to others; in this matter let every one judge and act for himself, and modify his practice according to his experience of its workings.

Whereas the hygrometric element, $\delta = \frac{3}{8}(\sqrt{ff'})$, acts a very important part in my new correct formulæ, I think it advisable to give the means of calculating it. I have given already in the Appendix, page 661, BESSEL'S excellent and extremely simple equation for calculating the elastic forces (or tensions) of vapour of water, corresponding to temperatures Centigrade, as also Fahrenheit thermometers. I will now give Dr. APJOHN'S formula for calculating the elastic forces or tensions of aqueous vapours of the atmosphere, from observations made with the wet and dry bulb hygrometers :—

DR. APJOHN'S *Formula for calculating the Elastic Force or Tension of the Vapour of Water of the Dew-point of the Atmosphere, from Observations made with the Wet and Dry Bulb Hygrometers.*

Let t be the observed temperature (Fahrenheit) of the dry thermometer.

Let t' be the observed temperature (Fahrenheit) of the wet thermometer.

Let f' be the elastic force (or tension) of the vapour of water, corresponding to the observed temperature t' of the wet thermometer, as ascertained from a sound table of elastic forces (or tensions) of aqueous vapour, or calculated by means of BESSEL'S equation, or otherwise correctly obtained.

Let F be the elastic force (or tension) of aqueous vapour, corresponding to the dew-point of the atmosphere, being extremely nearly equal to the actual elastic force (or tension) of the aqueous vapour of the atmosphere, and being the unknown quantity to be determined by the formula.

We have—

$$F = f' - 0{\cdot}01135\ (t - t') \times \frac{p - f'}{30} \text{ for temperatures above the freezing-point.}$$

$$F = f' - 0{\cdot}01014\ (t - t') \times \frac{p - f'}{30} \text{ for temperatures below the freezing-point.}$$

N. B.—The character p indicates the weight or pressure of the atmosphere in inches (as determined by a barometer, or otherwise determined accurately); the value of F is given in inches; and whereas $\frac{p - f'}{30}$ is seldom much different from unity, this quantity may *always* be omitted, when calculating the value of the hygrometric element $\{\delta = \frac{3}{8}\sqrt{(ff')}\}$.